Stabilization of reaction-diffusion pulses by external forcing

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A pulse velocity equation under forcing is derived and the conditions for stationary waves (pinning conditions) are considered. It is found that there are two types of stationary pulses with symmetric and asymmetric parameter sets in the phase-amplitude diagram. The pulses with a symmetric set are always unstable, whereas the pulses with an asymmetric set may be stable. The stability criteria are presented.

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I. PROBLEM STATEMENT

Recently [1], a pulse stabilization phenomenon by temporal forcing was reported. In our present research, we start from a different spatiotemporal situation, where the forcing $\overline{f}(x,t)$ is nonmoving in the comoving frame (comoving forcing), i.e., traveling with the wave, $\overline{f}(x,t)=\overline{f}(\xi)$, where $\xi=x-ct$ is a traveling wave coordinate and c is the wave velocity. This situation can be modeled experimentally as a spatially distributed light source which moves together with the propagating wave. When the wave is stopped (pinned by inhomogeneities) we just have spatial forcing. With this type of forcing, pinning and stability were studied for front waves in Ref. [2].

In a reaction-diffusion system under comoving forcing there is a family of wave solutions with different phases.[1] This is due to the fact that the translational invariance of the model equations is violated in the presence of the ξ dependent forcing and hence the phase value cannot be chosen arbitrarily. In other words, the wave cannot be centered at any point in space. In a preceding paper [3], we have studied the front velocity behavior under periodic comoving forcing of cosine type and now we will extend the analysis to pulse waves. To achieve this, we have to derive the velocity equation for pulse solutions and consider the zero velocity ($c=0$) case then. The wave solutions are the basis for the construction of the velocity equation, providing one quantity of interest, during the matching procedure. Setting $c=0$ in the velocity equation we obtain a pinning condition for the wave. The next step is as follows. Fronts and pulses in the unforced system have different stability features: the front is stable, whereas the pulse is unstable. Therefore, the essential task in our research is an exploration of the stabilization behavior of pulses in the presence of forcing.

II. VELOCITY EQUATION

We use the piecewise linear approximation for a cubic reaction term which is characterized by an inverted N-shaped dependence with three zeros [4]. At first, we do not specify a concrete form of the forcing; the only restriction is that the forcing must be small enough to avoid that the system jumps from one branch of the piecewise linear reaction term to the other one. This would lead to additional matching points and an increasing number of solution intervals to be taken into account. The considered reaction-diffusion equation under forcing reads

$$\frac{\partial u}{\partial t} = -u - 1 + 2\theta(u) + \overline{f}(x,t) + \frac{\partial^2 u}{\partial x^2}, \tag{1}$$

where $\theta(u)$ is the Heaviside function and the forcing $\overline{f}$ is a function of only ξ: $\overline{f}(x,t)=\overline{f}(\xi)$. The traveling wave equation obtained from Eq. (1) is a Duffing-Holmes type equation with a piecewise linear approximation of the cubic nonlinearity. There have been, of course, many articles studying oscillations in the Duffing-Holmes equation (e.g., Refs. [5]). This equation describes a particle moving in a double-well potential in the presence of friction, $V(u)\propto -u^2+u^4$, subject to a periodic force, where one identifies $u(\xi)$ with a spatial coordinate, ξ with time, and c with a friction coefficient. In our paper, we will construct traveling wave solutions. Due to the piecewise structure of the reaction term $f(u)=-u-1+2\theta(u)$, pulse solutions must consist of three pieces (i.e., there exist two matching points ξ_0 and ξ_0^*). Imposing the boundary conditions at infinity we obtain ($B=\text{const}$)

$$u_1(\xi) = B_1 e^{\lambda^+\xi} + \overline{u}(\xi) - 1, \; \xi \leqslant \xi_0,$$

$$u_2(\xi) = B_{21} e^{\lambda^+\xi} + B_{22} e^{\lambda^-\xi} + \overline{u}(\xi) + 1, \; \xi_0 \leqslant \xi \leqslant \xi_0^*, \tag{2}$$

$$u_3(\xi) = B_3 e^{\lambda^-\xi} + \overline{u}(\xi) - 1, \; \xi \geqslant \xi_0^*.$$

Here we take into consideration that the eigenvalue λ^+ is positive and λ^- is negative, $\lambda^\pm = -c/2 \pm \sqrt{c^2/4+1} \equiv -c/2 \pm \gamma$ are the eigenvalues of the homogeneous problem, and $\overline{u}(\xi)$ is a particular solution of the forced equation.

Reducing the number of matching equations (by elimination of the constants B) we obtain the velocity equation

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[1]The phase is representative of the wave position relative to the forcing positioning and is expressed in terms of a matching point coordinate for the piecewise linear model.

$$\lambda^- \ln\left(\frac{c/2 + \gamma\overline{u}(\xi_0)}{-\lambda^-}\right) + \lambda^+ \ln\left(\frac{-c/2 + \gamma\overline{u}(\xi_0^*)}{\lambda^+}\right) = 0. \quad (3)$$

This pulse velocity equation has a form similar to the front velocity equation in the tristable model [6]. The mathematical origin of this fact is that both waves have a three-piece structure with two matching points. In both cases one matching point value (for example, ξ_0^*) is determined by the matching conditions as a function of the other. The expression for the second matching point in the pulse case reads

$$\xi_0^* = \xi_0 + \frac{1}{\lambda^+} \ln\left(\frac{-\lambda^-}{c/2 + \gamma\overline{u}(\xi_0)}\right), \quad (4)$$

i.e., the matching point value ξ_0 is a free parameter; the wave solution depends on this value ξ_0.

When deriving Eqs. (3) and (4), it was noted that there are two restriction conditions

$$c/2 + \gamma\overline{u}(\xi_0) > 0 \text{ and } -c/2 + \gamma\overline{u}(\xi_0^*) > 0; \quad (5)$$

those are a mathematical consequence of the appearance of exponential terms in the matching equations and, hence, logarithms in the velocity equation. Their origin is not the forcing type but due to the existence of the second matching point. The next restriction is evident from the fact that ξ_0^* must be larger than ξ_0. Together with the first restriction in Eq. (5), this gives the interval of possible values for $\overline{u}(\xi_0)$:

$$-\frac{c}{2\gamma} < \overline{u}(\xi_0) < 1. \quad (6)$$

The appearance of these restrictions is not a particular feature of the forced system. The conditions (5) restrict the choice of the reaction function $f(u)$. It can be shown that for the unforced system with the general form $f(u) = -u - a_1 + (a_1 + a_2)\theta(u)$, where $a_{1,2}$ are constant, the same restrictions require asymmetry, $a_2 > a_1$, and when the function is symmetric, a pulse solution does not exist. This fact is well known in the bistable model (see [4]).

The origin of the restrictions can be explained using a particle-in-a-potential analogy for the traveling wave equation. The potential $V(u) \propto \int f(u)du$ is piecewise parabolic, and the pulse solution connects states with $u_1 = u_2 = -a_1$ as $\xi \to \pm\infty$. When the maximum of the a_2 branch is higher than the maximum of the a_1 zone, a pulse solution exists. Otherwise, there is no solution at all. In this paper we consider a system with a symmetric $f(u)$ but under forcing. The periodic forcing breaks the symmetry by moving the reaction function up and down in the $[u, f(u)]$ phase plane, causing changes in the excitability of the local dynamics as well as transitions between mono- and bistability [2].

Special attention must be paid to the first formula in Eq. (5). Comparison of this result with the front velocity equation $c/2 + \gamma\overline{u}(\xi_0) = 0$ [3] shows that the pulse velocity must be different from the front velocity; in the case of constant forcing at a positive value of the forcing amplitude, the pulse velocity is larger than that of the front. This fact can be explained again using the particle-in-a-potential analogy. In this analogy, c is a friction coefficient. Therefore, the particle returns to its original (initial) position at a lower local maximum (pulse solution) when the friction coefficient is large enough, whereas the front connects both maxima for smaller values of the coefficient of friction. Moreover, since the second maximum is higher (it is an absolute maximum), the friction coefficient must be negative in this situation.

III. STATIONARY WAVES

Now we will look more closely at stationary waves corresponding to a wave velocity equal to zero. The zeros of the velocity uniquely characterize the stationary solutions and in analogy to solid state physics we call these solutions pinned waves, the zeros of the velocity curve indicating pinning positions. Setting $c = 0$, it follows from Eq. (3) that the pulse is motionless when

$$\overline{u}(\xi_0)_{c=0} = \overline{u}(\xi_0^*)_{c=0}, \quad (7)$$

so that Eq. (4) transforms into

$$\xi_0^* = \xi_0 - \ln[\overline{u}(\xi_0)_{c=0}]. \quad (8)$$

From the result obtained, it may be concluded that stationary fronts and pulses exist for different parameter values because the front situation with $\overline{u}(\xi_0) = 0$ is eliminated from the pulse case due to the term $\ln[\overline{u}(\xi_0)]$ in ξ_0^*. When $\overline{u}(\xi_0) \to 0$ the second matching point tends to infinity, i.e., the pulse profile transforms into the front curve. So, the restriction conditions (5) and (6) are now reduced to

$$0 < \overline{u}(\xi_0)_{c=0} < 1 \text{ and } \overline{u}(\xi_0^*)_{c=0} > 0. \quad (9)$$

Equation (7) describes the condition when the pulse is stopped (pinning condition). Indeed, for the stationary case the comoving forcing degenerates into a spatial one, so that here the pinning effect (or, in other words, the propagation failure) is reproduced, meaning that a wave that is initiated in the medium cannot pass through the location of an inhomogeneity. Instead, it will be pinned at this location. This is best demonstrated when the inhomogeneity is abrupt or a step line [2]. In the case of periodic forcing there appear multiple pinning positions. To discuss this phenomenon in detail we consider a periodic forcing of cosine type,

$$\overline{f}(\xi) = f_0 \cos(\xi), \; f_0 = \text{const}. \quad (10)$$

Using the form (10), the particular solution $\overline{u}(\xi)$ of the forced equation becomes $\overline{u}(\xi) = R\cos(\xi) + Q\sin(\xi)$, where $R = 2f_0/(c^2+4)$, $Q = -cf_0/(c^2+4)$. For the stationary waves $Q = 0$, and $\overline{u}(\xi)$ has only one cosine term. Hence the pinning condition (7) yields

$$\cos(\xi_0) = \cos(\xi_0^*). \quad (11)$$

From the first restriction condition in (9) it follows that the pulse relationship (11) holds when $\cos(\xi_0) \neq 0$.

Using the expressions for ξ_0^* and $\overline{u}(\xi_0)$, the pinning condition (11) can be rewritten as a pair of relationships

$$\overline{u}(\xi_0)_{c=0} = \frac{f_0}{2}\cos(\xi_0) = e^{2(\xi_0 - \pi m)}, \quad (12a)$$

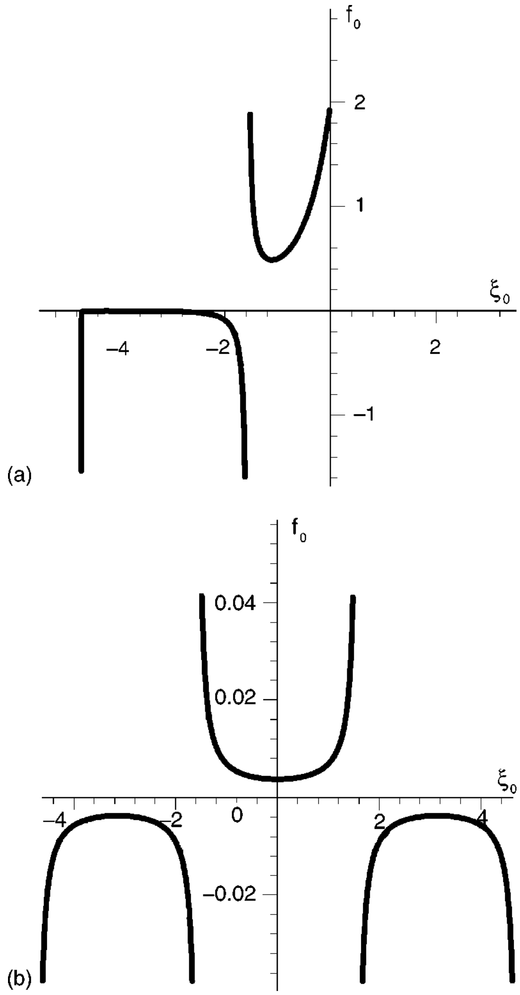

FIG. 1. Amplitude versus phase in the (ξ_0, f_0) plane. Diagram for stationary pulses under periodic forcing. The figure shows the forcing parameter plane where (a) the asymmetric [A set, Eq. (12a)] and (b) the symmetric [B set, Eq. (12b)] parameter sets are shown. The A set is displayed for $m=0$ and the B set with $l=1$.

$$\overline{u}(\xi_0)_{c=0} = \frac{f_0}{2}\cos(\xi_0) = e^{-2\pi l}. \tag{12b}$$

Due to the restrictions (9), the phase ξ_0 in Eq. (12a) limits to $\xi_0 < \pi m$, where $m=0, \pm 1, \pm 2, \ldots$ and in Eq. (12b) the restrictions yield $l=1,2,\ldots$ but the phase ξ_0 may be arbitrary.[2] The relationships (12) describe sets of curves that are asymmetric [Eq. (12a) and Fig. 1(a)] and symmetric [Eq. (12b) and Fig. 1(b)] with respect to the 0-f_0 axis on the (ξ_0, f_0) diagram. For the sake of brevity, we denote solutions satisfying Eqs. (12a) and (12b) as belonging to A and B sets, respectively. Figure 1 shows only the first two examples, at $m=0$ and $l=1$, of the dependencies (12), because other cases with $m \neq 0$ and $l > 1$ may be reduced to these examples by rescaling the forcing amplitude by factors $e^{\pm 2\pi}$. The symmetric solution (12b) is periodic in ξ_0 and there is a minimal forcing amplitude $f_0^{\min}$, so that no pulses exist at $|f_0| < f_0^{\min}$. However, in the general case when $l=1,2,\ldots$, infinite sets of solutions appear and the minimum $f_0^{\min}$ tends to zero. The asymmetric solution (12a) at $m=0$ holds only when the phase ξ_0 is negative. Inserting Eq. (12) into Eq. (8) we obtain

$$\xi_0^* = -\xi_0 + 2\pi m, \tag{13a}$$

$$\xi_0^* = \xi_0 + 2\pi l \tag{13b}$$

for the A set (12a) and B set (12b), respectively. In particular, for the (asymmetric) A set at $m=0$ both matching points are placed symmetrically about zero, $\xi_0^* = -\xi_0$, whereas for the B set at $l=1$ the second matching point is shifted by 2π from the first one.

IV. STABILITY CRITERIA

The question now arises whether these pulse solutions are stable. The details of the linear stability analysis of the front solutions in forced (inhomogeneous) reaction-diffusion equations can be found, for example, in Ref. [2]. To investigate stability, one considers a perturbed solution of the form $u(\xi) + \tilde{u}(\xi)e^{\omega t}$, where $u(\xi)$ is the unperturbed solution, $\tilde{u}(\xi)$ is a small perturbation, and ω is its growth rate. The linearized variational equation for the perturbation reads

$$\frac{d^2\tilde{u}}{d\xi^2} + c\frac{d\tilde{u}}{d\xi} - [1 + \omega - 2\delta(u)]\tilde{u} = 0, \tag{14}$$

where $\delta(u)$ is the Dirac delta function. By satisfying the boundary conditions the perturbation eigenfunction is obtained as

$$\begin{aligned} \tilde{u}_1(\xi) &= \tilde{B}_1 e^{\tilde{\lambda}^+ \xi}, \ \xi \leq \xi_0, \\ \tilde{u}_2(\xi) &= \tilde{B}_{21} e^{\tilde{\lambda}^+ \xi} + \tilde{B}_{22} e^{\tilde{\lambda}^- \xi}, \ \xi_0 \leq \xi \leq \xi_0^*, \\ \tilde{u}_3(\xi) &= \tilde{B}_3 e^{\tilde{\lambda}^- \xi}, \ \xi \geq \xi_0^*, \end{aligned} \tag{15}$$

where the signs of $\tilde{\lambda}^{\pm} = -c/2 \pm \sqrt{c^2/4 + 1 + \omega} \equiv -c/2 \pm \tilde{\gamma}$ are taken into account.[3] Computing the determinant of the matrix at $(\tilde{B}_1, \tilde{B}_{21}, \tilde{B}_{22}, \tilde{B}_3)$ for the system of matching equations we obtain the growth rate equation. In the stationary case, it reads

$$(\tilde{\gamma}\chi - 1)(\tilde{\gamma}\chi^* - 1) = [\overline{u}(\xi_0)_{c=0}]^{2\tilde{\gamma}}, \tag{16}$$

where $\tilde{\gamma} = \sqrt{1+\omega}$ and the jump parameters χ and χ^* are given by

$$\chi = \left| 1 - \overline{u}(\xi_0)_{c=0} + \left. \frac{d\overline{u}(\xi)_{c=0}}{d\xi} \right|_{\xi=\xi_0} \right|,$$

[2]Except for the forbidden situation when $\cos(\xi_0)=0$. This is true for both cases Eqs. (12a) and (12b).

[3]Here we consider the case of the growth rate ω having positive or small negative values, so that $1+\omega > 0$.

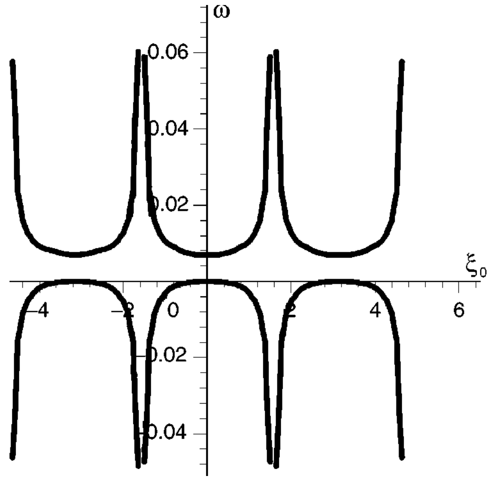

FIG. 2. Growth rate dependence $\omega=\omega(\xi_0)$ for stationary pulses under periodic forcing with the parameter set (12b). The curves in the diagram are described by Eqs. (16) and (19) with $l=1$ and plotted in the interval $-3\pi/2<\xi_0<3\pi/2$.

$$\chi^* = \left| 1 - \overline{u}(\xi_0)_{c=0} - \left.\frac{d\overline{u}(\xi)_{c=0}}{d\xi}\right|_{\xi=\xi_0^*} \right| . \qquad (17)$$

Here the pinning condition (7) has been taken into consideration. From Eqs. (17), it follows that the jump parameters χ and χ^* are the same for constant forcing.

Now, to be specific, we use the periodic force (10). Then $\overline{u}(\xi)_{c=0}=(f_0/2)\cos(\xi)$, and one can see from Eq. (13a) that for the A set both jump parameters are equal, $\chi^*=\chi$, and hence the growth rate equation (16) reads

$$\tilde{\gamma}\chi - 1 = \pm\left(\frac{f_0}{2}\cos(\xi_0)\right)^{\tilde{\gamma}} \qquad (18)$$

with $\chi=|1-(f_0/2)[\cos(\xi_0)+\sin(\xi_0)]|$. For the B set [see Eq. (13b)], the factors χ and χ^* are different,

$$\chi = \left| 1 - \frac{f_0}{2}[\cos(\xi_0)+\sin(\xi_0)] \right| ,$$

$$\chi^* = \left| 1 - \frac{f_0}{2}[\cos(\xi_0)-\sin(\xi_0)] \right| , \qquad (19)$$

and Eq. (16) keeps its form. However, in the particular case when $\xi_0=\pi n$, $n=0,\pm1,\pm2,\ldots$, we have $\chi^*=\chi$ and Eq. (16) reduces to Eq. (18) for the B set as well.

The growth rate equations (16), (19), and (18) are presented graphically in Figs. 2 and 3, respectively. The curves are plotted for the interval $-3\pi/2<\xi_0<3\pi/2$. The diagram in Fig. 2 shows that there are curves $\omega=\omega(\xi_0)$ for positive and negative values of the growth rate ω. Therefore, there are no stable pulses under forcing with the B set. The stability of the pulses with the A set is especially noteworthy. In this situation, there exist two curves on each interval (Fig. 3). Each curve corresponds to a solution of Eq. (18) with a different sign on the right side. The equation with positive sign describes the upper curve; the lower curve corresponds to Eq. (18) with negative sign. In the interval (a) $-\pi/2<\xi_0<0$ only the "plus sign curve" intersects the 0-ξ_0 axis [Fig. 3(a)], whereas on the interval (b) $-3\pi/2<\xi_0<-\pi/2$ both curves intersect this axis [Fig. 3(b)]. Since in the last case the curves increase with growth of the phase ξ_0, the stability behavior is crucially defined in both cases (a) and (b) by the plus sign curve. In each interval (a) and (b) (see Fig. 3), the A set has two regions: the first region at $-\pi(2k+1)/2<\xi_0<\xi_{\text{crit}}^{(k)}$, $k=0,1$, where the pulse is stable, and the second region at $\xi_{\text{crit}}^{(k)}<\xi_0<-\pi k/2$, where the pulse is unstable. To determine the values of the critical points $\xi_{\text{crit}}^{(k)}$ we set $\omega=0$ in Eq. (18) with plus sign. Then $\tilde{\gamma}=1$ and we obtain the equation

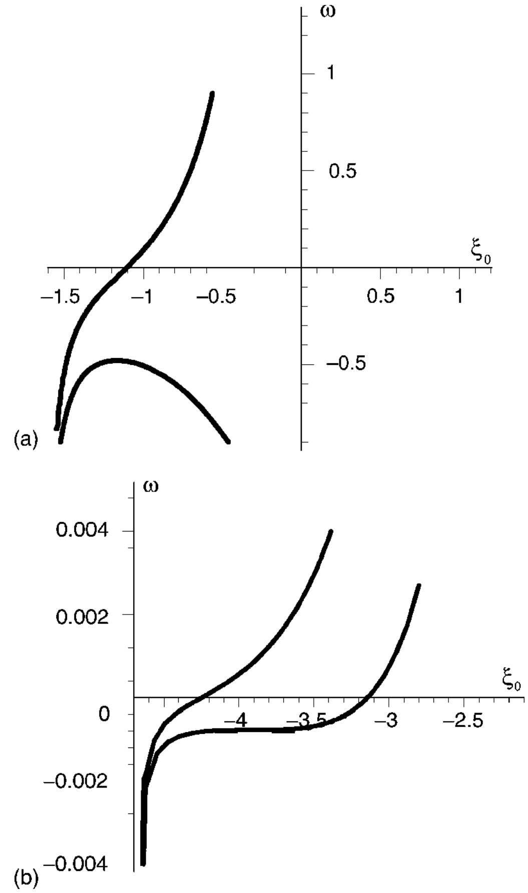

FIG. 3. Growth rate dependence $\omega=\omega(\xi_0)$ for stationary pulses under periodic forcing with the set (12a). The curves in the diagram are described by Eq. (18) with $m=0$ and plotted in the intervals (a) $-\pi/2<\xi_0<0$ and (b) $-3\pi/2<\xi_0<-\pi/2$. The upper curve corresponds to the plus sign on the right side of Eq. (18) and the lower one to the minus sign.

$$\left| 1 - \frac{f_0}{2}[\cos(\xi_0)+\sin(\xi_0)] \right| = 1 + \frac{f_0}{2}\cos(\xi_0), \qquad (20)$$

which has two solutions: $\tan(\xi_0)+2=0$ and $(f_0/2)\sin(\xi_0)-2=0$. For the second case, however, which is valid when $|f_0|>4$, there appear more than two matching points for the pulse profile, because the particular solution $\overline{u}(\xi)=(f_0/2)\cos(\xi)$ may then have a magnitude larger than unity and the whole solution $u(\xi)$ intersects the 0-ξ axis more than

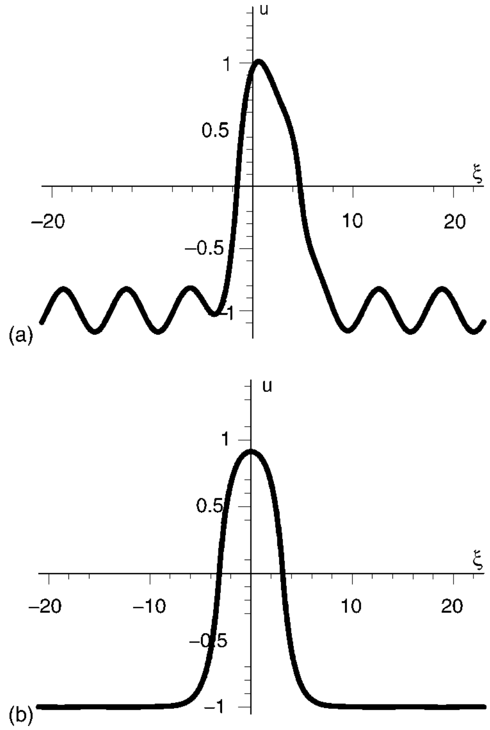

FIG. 4. Pulse profiles with the set (12b) when (a) $\xi_0=-1.56$ and (b) $\xi_0=-\pi$. The case (b) illustrates also a pulse with the A set [at this ξ_0 value the expressions (12a) and (12b) yield the same result]. Both pulses are unstable.

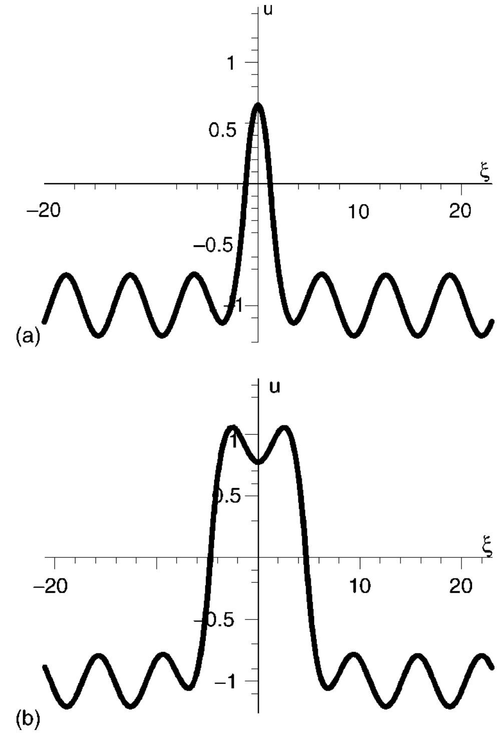

FIG. 5. Pulse profiles with the parameter set (12a) when (a) $\xi_0=-1.2$ and (b) $\xi_0=-4.712$. Both pulses are stable.

twice.[4] This situation is beyond the scope of the present problem. Therefore, the appropriate solution is only the first one, $\xi_{\text{crit}}^{(k)}=-\arctan(2)\approx-1.11,-4.25$ for $k=0,1$, respectively. These values are precisely the same positions as those of the local extrema of the ξ_0-f_0 curve in Fig. 1(a) in the intervals (a) and (b). Indeed, from Eq. (12a) it follows that $df_0(\xi_0)/d\xi_0=0$ when $\xi_0=-\arctan(2)$.

All of the preceding is intended to illustrate the behavior of the waves under forcing and explore the stabilization criteria. Now it is instructive to present the concrete pulse profiles. They are exhibited in Figs. 4 and 5 for the B set and A set, respectively. In both cases the pulse profile may be symmetric or asymmetric to the 0-u axis. But for $m=0$ the pulses corresponding to the A set have symmetric profiles. The pulses for the B set are usually asymmetric, and there is only one specific phase value, $\xi_0=-\pi l$, when the pulse with the B set is symmetric. In this case, both equations (12a) and (12b) give the same forcing amplitude, i.e., both pulse solutions coincide. The pulse for this situation is displayed in Fig. 4(b) for $l=1$. The value of the forcing amplitude is extremely small and the oscillations in the wave profile are difficult if not impossible to detect. There are pulses with one [Fig. 5(a)] and many crests. The many-crest pulses [Fig. 5(b)] can appear for both sets, which requires a large value of the phase difference $\Delta\xi=\xi_0^*-\xi_0$. The stable and unstable pulse profiles are similar, so that in Fig. 5 only stable pulses are depicted.

[4]We are reminded here that the fixed points of the unforced system are placed at $u(\xi\to\pm\infty)=-1$.

V. CONCLUDING REMARKS

The wave stabilization phenomenon takes place for many forcing functions. For example, we have also considered a simple polynomial function $\overline{f(\xi)}=f_0-\xi^2$, $f_0=\text{const}$, as forcing. The minus sign and the quadratic function[5] here are essential to provide only two matching points in the pulse profile and the f_0 term—to realize (satisfy) the restriction condition (9). The particular solution reads $\overline{u(\xi)}=f_0-\xi^2-2(c^2+c\xi+1)$, and the pinning condition (7) yields $\xi_0^*=-\xi_0$, i.e., the stationary wave profile is symmetric about the 0-u axis. For this type of solution, the jump parameters χ and χ^* are equal to one another and the growth rate equation takes the form like Eq. (18)

$$\tilde{\gamma}\chi-1=\pm[f_0-\xi_0^2-2]^{\tilde{\gamma}}, \tag{21}$$

with $\chi=|3-f_0-2\xi_0+\xi_0^2|$. From Eq. (21), one may find that, for example, when the forcing parameter is $f_0=2.5$, the pulse is stable for the phases $\xi_0=-0.7,\ldots,-0.64$ (the interval of validity is constrained by $-1/\sqrt{2}<\xi_0<0$).[6]

In conclusion, we emphasize the main result of the work: *there exist forcing parameter intervals such that the pulse wave can be stabilized by periodic external forcing.* We have

[5]Here we still investigate the stability of the pulse solution. If we consider the front wave, then the simplest polynomial function is a linear ramp.

[6]This is just an illustration. We did not perform the regular computations and the analysis of curves.

demonstrated this result for the stationary case, i.e., for the spatial forcing. Since the pulse may be interpreted as a bound pair of fronts and then the pulse stability depends on the interaction between these fronts [1], the aim of our new research is to construct a pair of fronts moving with different velocities or, in other words, a pulse with growing or shrinking profile. The calculations will illustrate how waves (front and back) interact during collisions, and how they are influenced by local distrurbances. There exist many interesting phenomena in periodically forced reaction-diffusion systems to be explored [7].

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